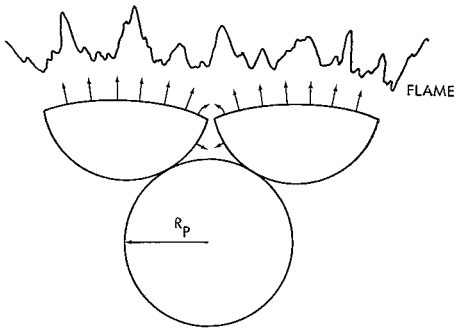

Fig 1 Schematic of particle initiation process

the effective initiation flux varied inversely with particle diameter Equation (2) can then be written in the form

$$t_i = kd^{1.8} \tag{3}$$

Once the combustion of the particle has been initiated, it is assumed that the particle burns at a constant linear rate The particle-burning time can thus be defined by

$$t_b = k_1 d \tag{4}$$

Substituting Eqs (3) and (4) into (1), the total time is then given by

$$\tau = kd^{1.8} + k_1 d \tag{5}$$

The experimentally observed burning velocity is defined as

$$v_a = d/\tau \tag{6}$$

Replacing τ by Eq (5) and rearranging to show particle size dependence more clearly, Eq (6) becomes

$$1/v_a = kd^{0.8} + k_1 \tag{7}$$

An analytical expression for k_1 can be obtained from an approximate solution[4] of the energy equation describing the over-all deflagration process

In terms of the proposed model, the first term on the right in Eq (7) shows the influence of the initiation process on the observed burning rate, whereas the second term is the actual particle-burning velocity independent of initiation factors If the foregoing relationship is indeed capable of representing the experimental data, a family of curves with pressure as a parameter would result at each initial solid temperature

Figures 2 and 3 show the experimental data obtained in Ref 4 plotted in the form indicated by Eq (7) A Crawford strand burner with hot wire ignition was used to obtain the experimental data The solid lines in Figs 2 and 3 were obtained by applying the technique of least mean squares to the experimental data It is apparent that the burning rate can be

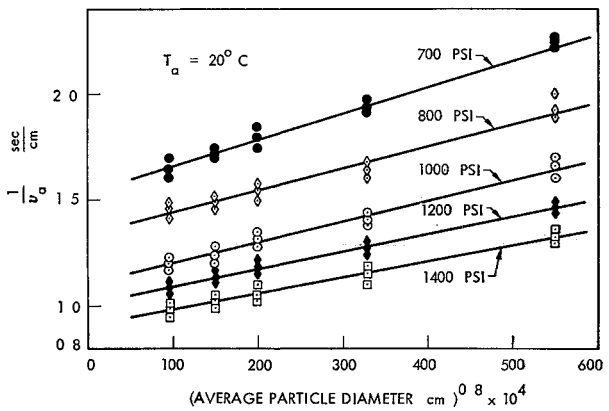

Fig 2 Strand burning rate as function of particle size—20°C

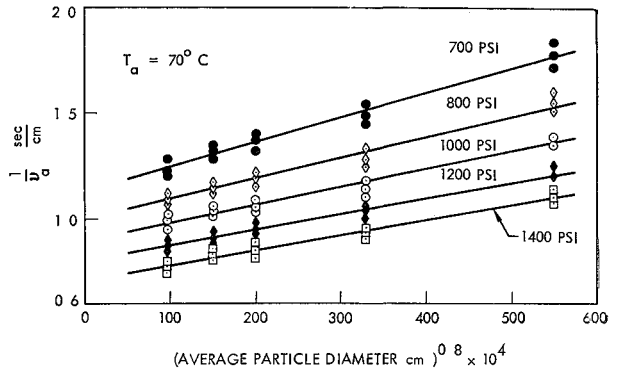

Fig 3 Strand burning rate as function of particle size—70°C

adequately correlated, in the particle-size range studied in this investigation, by the use of Eq (7)

The ability of the model to describe the effects of particle size, pressure, and strand density on the burning rate lends support to the idea that the deflagration of ammonium perchlorate strands can be characterized as being the result of the propagation of a continuous ignition front In addition, the model clearly indicates the direction for future work in this area In order to develop a quantitative description of ammonium perchlorate deflagration, the several unit mechanisms occurring must be investigated in greater detail Specifically, the more important unknown areas requiring study are the nature and magnitude of the kinetic reaction processes associated with the particle-ignition phenomena and the burning rate of the individual particles

References

[1] Bircumshaw, L L and Newman, B H , "The thermal decomposition of ammonium perchlorate," Proc Roy Soc (London) **A227**, 115–132 (1954); **A227**, 228–241 (1955)

[2] Friedman, R and Levy, J B , "Research on solid-propellant combustion," Atlantic Res Final TR, Air Force Office Sci Res 2005 (1961)

[3] Arden, E A , Powling, J , and Smith, W A W , "Observations on the burning of ammonium perchlorate," Combust Flame **6**, 21–33 (1962)

[4] Shannon, L J , "Effects of particle size and initial solid temperature on the deflagration rate of ammonium perchlorate strands," Ph D Thesis, Univ Calif (1963)

Heat Transfer from Nonequilibrium Ionized Argon Gas

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THE effect of ionization on boundary-layer flow and heat transfer has been a matter of considerable argument in recent years, for, apart from the scarcity of experimental data, a discrepancy was found between the result obtained by Scala and Warren[1] and those by others which are summarized in Refs 2 and 3 In the present work, the effect of ionization on heat transfer is investigated for argon flow both theoretically and experimentally

The transport properties of argon used for the calculation is that of Penski,[4] in which the transport properties are determined using the Chapman-Enskog method from the available data on electron scattering intensity The electron-

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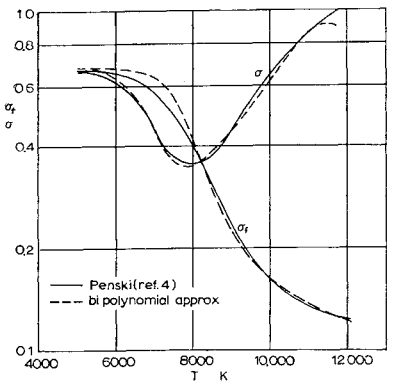

Fig 1 Equilibrium and frozen Prandtl numbers of ionized argon calculated by Penski at 0 1 atm and their analytic approximation

atom collision cross section in this method varies from about $1\,6 \times 10^{-16}$ cm^2 at 7000°K to about 3×10^{-16} cm^2 at 12,000°K at 0 1 atm The frozen and equilibrium Prandtl numbers obtained by this method are shown in Fig 1 The equilibrium composition is determined by Saha equation [5]

In the theoretical analysis,[7] the equations are set up for the following dimensionless variables:

$$f' = \frac{u}{u} \qquad g = \frac{h}{h_e} \qquad \phi = \frac{N_i}{N_a + N_i}$$

where u is the velocity along the surface, h the frozen total enthalpy $h = 2\,5(1 + \phi)RT + \frac{1}{2}u^2$, the subscript e denotes the edge of boundary layer, N is the number density, and the subscripts a and i refer to the neutral atoms and ions, respectively The gas is assumed to be a binary mixture of heavy particles, i e , atoms and ions, and electrons The equations of motion can be set up as in Ref 2 and become, for the frozen flow having similar profiles,

$$\left(\frac{C}{S}\phi'\right)' + f\phi' = 0 \tag{1}$$

$$(Cf'')' + ff'' + \beta\left(\frac{\rho_e}{\rho} - f'^2\right) = 0 \tag{2}$$

$$\left[\frac{C}{\sigma_f}\{g' + (L - 1)g\phi'\} + \frac{u_e^2}{h}f'f''\left(1 - \frac{1}{\sigma_f}\right)\right]' + fg' = 0 \tag{3}$$

In these equations C is the viscosity-density factor $C = \rho\mu/(\rho\mu)$, S and L the effective Schmidt and Lewis numbers for electron diffusion, σ_f the frozen Prandtl number, and β is zero for flat plate and $\frac{1}{2}$ for axisymmetric stagnation point For an equilibrium flow, Eq (3) is reduced to[6]

$$\left\{\frac{C}{\sigma}g' + \frac{u_e^2}{h}f'f''\left(1 - \frac{1}{\sigma_f}\right)\right\}' + fg' = 0 \tag{4}$$

where σ is the equilibrium Prandtl number

Equations (1–4) are solved with the appropriate boundary conditions with the condition $\phi(0) = 0$, i e , infinitely catalytic surface, for the axisymmetric stagnation point and the flat-plate flows The frozen Prandtl number is calculated assuming that it is a function only of ionization fraction The equilibrium Prandtl number is taken for the pressure of 0 1 atm The effective Schmidt number for electron diffusion is taken as $\frac{1}{2}$, which is based on the assumption that the mean free path of ions is the same as that of neutral atoms and that the Schmidt number for diffusion of atoms is unity The viscosity-density factor C is approximated by the inverse square root of temperature ratio The solutions are obtained through iteration using the Ferranti Mercury digital computer The results are shown in Fig 2

To check the theory, the over-all heat transfer to a hemisphere and a 30° included-angle cone has been measured in

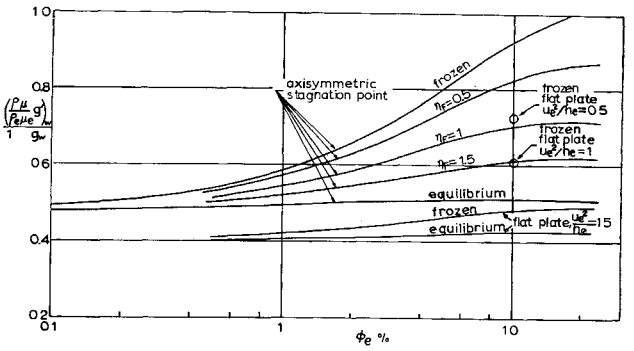

Fig 2 Computed heat-transfer parameters for ionized argon for equilibrium, frozen, and suddenly freezing boundary layers In a suddenly freezing boundary layer, the flow is in equilibrium beneath the point $\eta = \eta_f$ and frozen beyond it

the Imperial College 1-in plasmajet wind tunnel[8] with argon as the working gas The steady-state heat-transfer rate was measured using a water-cooled model of 0 4-in diam The surfaces used were copper and copper coated with a borosilicate ceramic material. The settling chamber pressure varied from about 0 2 to 0 3 atm The nozzle exit area ratio was about 3 5 The maximum enthalpy level reached was about 5000 Btu/lb, giving a calculated ionization fraction of about 8% The heat-transfer results are presented in Fig 3

The calculation of the test flow conditions[7] is based on the measurements of the energy conversion efficiency of the plasmajet head and of the settling chamber pressure and mass flow rate A detailed calculation of displacement thickness at the throat and the convective heat loss to the nozzle wall is carried out using the method of Ref 9 to compare the two methods, as well as the exact computation of ionic relaxation in the nozzle The displacement thickness at the throat was negative and was equal in magnitude to about 1% of the throat diameter, which contributed to increasing the frozen enthalpy by 8% in the calculation The heat loss to the nozzle wall from the point at which the Mach number is 0 3 onward was about 3% of the total electrical energy input to the head The relaxation time for the equilibrium of electron temperature with the atom temperature is checked for the flow in the tunnel by using the method of Ref 10 and was found to be very small compared with the mechanical transit time Therefore, the electron temperature was assumed to be equal to the atom temperature, neglecting the radiation loss The ionic recombination process through the nozzle was computed using the recombination rate of Ref 11 using a

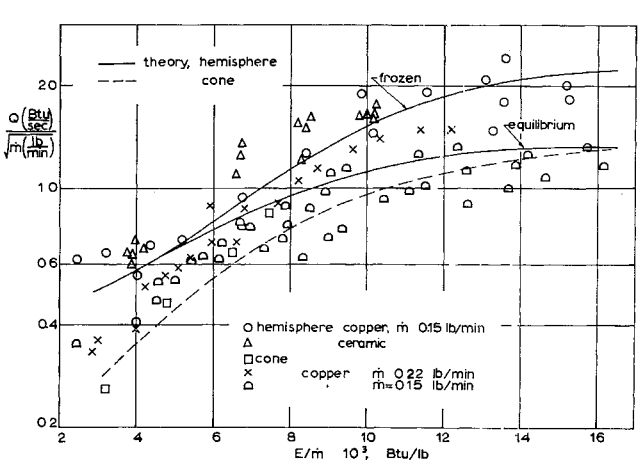

Fig 3 Experimental results of heat transfer from ionized argon (E = electrical energy input to plasma jet head, Q = over-all heat-transfer rate to model, and $\dot{m}$ = mass flow rate through nozzle) The theoretical curve for cone is the mean of frozen and equilibrium values

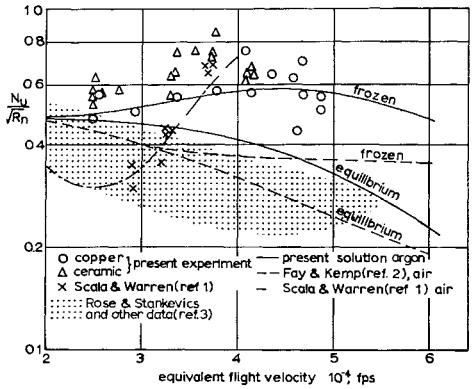

Fig 4 Comparison of heat-transfer results with other data; stagnation-point heat transfer (N_u = Nusselt number, R_n = Reynolds number)

program that is an improved version of the one used in Ref 12 which solves the flow upstream of the throat as well as downstream Significant ionic recombination was predicted by this method The enthalpy levels calculated using the two methods just described agree within the accuracy of measurement (±25%) over the whole operating range

The formula in Ref 13 was used to calculate the local heat-transfer rate over the hemisphere The resulting theoretical heat-transfer rates are shown in Fig 3 As seen from the figure, there is no difference between the heat transfer to the copper and ceramic surface The heat transfer to the cone shows good agreement between theory and experiment The heat-transfer rate to the hemisphere shows that the boundary-layer flow at the stagnation region is nearly frozen The examination of the flow Damköhler number in the boundary layer shows that the flow should be ionically frozen except for the immediate vicinity of the wall Thus, the theory and experiment both support the conclusion that the boundary-layer flow under these conditions is nearly frozen

In order to compare the present result with equivalent data for air, the heat-transfer rate to the hemisphere is plotted in Fig 4 with the equivalent flight speed as abscissa The equivalent flight speed was calculated to give the same enthalpy fraction, i e , the ratio of energy consumed in ionizing to the total energy, as in flight The difference between the experimental results presented here and those of Rose and Stankevics and other similar ones summarized in Ref 3 can be attributed to the difference in test conditions In Ref 3 the flow was in equilibrium; here the flow is nearly frozen The test conditions relevant to Ref 1 are unknown, but from the foregoing it seems that the flow must have been frozen The disagreement between the results of Scala and Warren[1] and the theory of Ref 2 for frozen flow remains unresolved Figure 4 also shows that the difference between the theoretical heat transfer from a frozen and equilibrium boundary layer is more severe for argon than for air This may be because the electron-atom collision cross section in argon is smaller than in air, and hence the frozen Prandtl number decreases more rapidly as the ionization fraction increases than in air

References

[1] Scala, S M and Warren, W R , "Hypervelocity stagnation point heat transfer," ARS J **32**, 101–102 (1962)

[2] Fay, J A and Kemp, N H , "Theory of stagnation point heat transfer in partially ionized air," IAS Paper 63-60 (1963)

[3] Rose, P H and Stankevics, J O , "Stagnation point heat transfer measurement in partially ionized air," IAS Paper 63-61 (1963)

[4] Penski, K , "Zustands- und Transportgrossen von Argonplasma," Chem Ing -Tech **34**, 84-87 (1962)

[5] Bray, K N C and Wilson, J A , "A preliminary study of ionic recombination of argon in wind tunnel nozzle," Univ Southampton, Aeronaut. and Astronaut Rept 134 (1960)

[6] Cohen, N B , "Boundary layer similar solutions and correlation equations for laminar heat transfer distribution in equilibrium air at velocities up to 41,100 feet per second," NASA TR R 118 (1961)

[7] Park, C , "Some aspects of chemical nonequilibrium in high temperature high speed flows,' Ph D Thesis, Univ London, Imperial College (1964)

[8] Harvey, J K and Simpkins, P G , "A description of the Imperial College arc-heated wind tunnel, ' J Roy Aeronaut Soc. **66**, 637–641 (1962)

[9] Cohen, C B and Reshotko, E , "The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient," NACA TN 3326 (1955)

[10] Spitzer, L , Jr , *Physics of Fully Ionized Gases* (Interscience Publishers, New York, 1956), Chap 5

[11] Hinnov, E and Hirschberg J G , "Electron-ion recombination in dense plasmas," Phys Rev **125**, 795–802 (1952)

[12] Stollery, J L and Park, C , "Computer solutions to the problem of vibrational relaxation in hypersonic nozzle flows," Imperial College Aeronaut Rept 115 (1963)

[13] Kemp, N H , Rose, P H , and Detra, R W , "Laminar heat transfer around blunt bodies in dissociated air" J Aerospace Sci **26**, 421–430 (1959)

Effective Heat of Ablation of Graphite

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THE effective heat of ablation, designated Q^* or H_{eff}, is an artifice often used for the prediction or comparison of the performance of ablation materials It is usually defined as[1]

$$Q^* = q_0/\dot{m} \tag{1}$$

where $\dot{m}$ is the mass ablation rate and q_0 is the heat-transfer rate to a nonablating surface which is at the ablation temperature

$$q_0 = (g\rho_e u\, c_{f\,0}/2)(h_{se} - h_w) \tag{2}$$

where ρ_e, u , and h_s are, respectively, the density, velocity, and total enthalpy at the edge of the boundary layer; h_{ew} is the enthalpy of the gas that exists at the edge of the boundary layer (air or other medium) evaluated at surface conditions; and $c_{f\,0}$ is the skin-friction coefficient in the absence of ablation With this definition, Q^* is the energy absorbed or blocked per unit mass of material ablated Hence, Q^* is usually considered as a measure of the effectiveness of the ablation material

Graphite does not go through a melting phase, and sublimation does not occur until very high surface temperatures are attained At lower temperatures, ablation of graphite in air occurs because of chemical reaction with oxygen Combustion rates are relatively low, however, so that the heat blockage as a result of mass addition to the boundary layer is minor The heat that is conducted to or radiated from the graphite can actually be increased by the combined effects of the exothermic chemical reaction and the mass transfer, whereas it is markedly reduced for most ablation materials such as the reinforced plastics, fused silica and teflon Because of the low ablation rates, the effective heat of ablation of graphite is much higher than that of most other ablators, considerable heat being stored in the solid phase and reradiated from the surface There has been considerable misunderstanding regarding the heat of ablation of graphite The purpose of this note is to attempt to clear up some of this confusion and to bring to attention a more convenient method for the rapid estimation of graphite ablation rates [2]

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